MULTI-CRITERIA FUCOM-MAIRCA MODEL FOR THE EVALUATION OF LEVEL CROSSINGS: CASE STUDY IN THE REPUBLIC OF SERBIA

Dragan Pamučar*, Vesko Lukovac, Darko Božanić, Nenad Komazec

University of Defence in Belgrade, Military Academy, Belgrade, Serbia

Received: 14 October 2018
Accepted: 07 December 2018
Published: 19 December 2018

Abstract: A level crossing, as a point of the crossing of road and rail traffic in the same level, is a place of conflicts subject to traffic accidents. In Serbia, the selection of the level crossings to be secured is mostly done based on the media and society pressure, as a result of an increase of the number of accidents at level crossings. This paper presents the application of a group multi-criteria FUCOM-MAIRCA (Full Consistency Method – Multi Attributive Ideal-Real Comparative Analysis) model that supports the process of selecting a level crossing in terms of investing in its security equipment. The FUCOM-MAIRCA multi-criteria model is tested in a case study which included the evaluation of ten level crossings within the railway infrastructure in the Republic of Serbia. The evaluation of the crossings is carried out through the assessment according to seven criteria set out on the basis of representative literature and surveys of experts. Sensitivity test of the FUCOM-MAIRCA model is performed by changing the weight coefficients of criteria and statistically processing the results using Spearman’s rank correlation coefficient.

Key words: railway level crossings, FUCOM, MAIRCA, multi-criteria decision making, railway accidents.

1. Introduction

A level crossing, as a point of the crossing of road and rail traffic in the same level, is a place of conflicts subject to traffic accidents (Law on Road Traffic Safety, 2018), which can have the consequences in terms of material damage and/or perished persons (Pamučar et al., 2015). At the occurrence of traffic accident between the road and the rail vehicle, there is an exchange of collision forces that are extremely high due to large mass of the two vehicles. The contact during the accident is usually made between the front part of a train and the lateral part of a road vehicle,
so that such traffic accidents often result in substantial material damage or severe injuries.

It is estimated that as an average 1.312 lives are lost per day in traffic accident in the world (Park, 2007). According to the report of the European Railway Agency, 27% of total number of deaths in the railroad accidents happens at level crossings (Ćirović & Pamučar, 2013). Traffic accidents at level crossings are mostly the result of misconduct and careless behavior of participants in road traffic. In the previous year (2017) in Serbia in 57 accidents, eight people are died that shows the significance of this problem. Such statistics are not only in Serbia, but the indicators are approximate in other countries, and this problem has been recognized by the International Rail Union.

According to the EU statistics (European Railway Agency, 2011), the volume of rail transport will be doubled over the following 30 years, which is a direct indication of the expected increase of extraordinary events at level crossings on all railroads, including those in Serbia. Increasing traffic volume will increase also the need for raising the level of crossing insurance at road crossings. In this context, the need for investing funds in terms of road crossings safety will also be defined. The provision of road crossings with modern safety equipment is costly investment, so when making an investment decision, the responsibility of the management is high, because the approved funds have to provide adequate effect.

In Serbia, 77% of level crossings are not secured according to the Law on Traffic Safety and applicable instructions of the Serbian Railways (Pamučar et al., 2015). On the Serbian railways network with a total length of 6,974 km there are 2354 level crossings, 108 of which are pedestrian crossings. Out of this number, 588 crossings are secured with automatic or mechanical devices. Securing level crossings represents significant material expenditure, so it is necessary to be grounded on reliable strategies for the selection of a level crossing that needs to be secured, as well as to be supported by the investments realization plan in terms of its security.

In this paper is proposed the FUCOM-MAIRCA multi-criteria model for the evaluation of level crossings and the creation of a strategy for the selection of priority level crossings that need to be secured. The criteria affecting the selection of the level crossing for the installation of necessary equipment to increase the security are defined. The survey of experts is conducted in the research in order to collect necessary data for determining relative weight criteria using the FUCOM model. The final evaluation and selection of priority crossings is carried out using the MAIRCA model.

Through the research and development of the model in this paper several goals are presented: (1) Review of the existing methodologies for the evaluation of level crossings; (2) Improving the methodology for crossings evaluation and selecting priority crossings for the installation of security equipment through the development of original multi-criteria FUCOM-MAIRCA model; (3) Proposal of new methodology for the identification of high-risk crossings; (4) Bridging the gap that currently exists in the methodology for evaluating and selecting priority crossings for the installation of safety equipment; and (5) Popularizing and affirming new models of multi-criteria
decision making (FUCOM and MAIRCA models) through their application in making complex decisions.

The remaining sections of the paper are organized as follows. In the second part, a brief overview of the literature is presented and a review of similar research topics in which are applied the models for the selection and evaluation of crossings. In the third part, the models used are briefly presented and the FUCOM-MAIRCA algorithm is shown. The fourth part presents a case study in which is carried out the evaluation of ten railway crossings within the railway infrastructure in the Republic of Serbia. The fifth part includes the sensitivity analysis in terms of testing the stability of the results by changing the weight coefficients of criteria in the FUCOM-MAIRCA model. The sixth part presents key contributions of the developed model, as well as suggestions for future research.

2. Literature review

The first mathematical models for the evaluation and ranking of crossings were developed in the mid 20th century (Berg, 1966). Berg (1966) presented the model for the evaluation and ranking of level crossings on the basis of a statistical model for predicting the number of traffic accidents. Later Qureshi et al. (2003) improved the statistical model shown by Berg (1966) through the application of data mining. In addition to the above mentioned models, in many countries of the world the evaluation of crossings is performed using Quantified Risk Analysis (QRA). QRA provides a suitable basis for establishing level crossing improvement priorities. This it does by allowing a ranking of level crossings in terms of their accident risk probability. Those crossings with high accident probabilities would normally qualify for funding allocations, while those with low accident probabilities would be assigned a low priority for improvement funding. The Oregon State Highway Department completed a study concerned with measuring the relative hazards of railroad grade crossings located on state and federal-aid highway systems (Tey et al., 2009). The majority of the 400 grade crossings considered were located in incorporated areas. Application of QRA can be see in papers of many authors (Reiff et al., 2003; Tey et al., 2009; Anandarao & Martland, 1998; Woods et al., 2008).

The Armour Research Foundation has conducted two grade crossing accident studies for the Association of American Railroads results of an analysis of 2,291 grade crossings in the State of Iowa were reported in 1958 (Crecink, 1958). Regression analysis techniques were utilized to develop risk factors (the expected accident rates at grade crossings over a 16-year period) as a function of type of protection, highway traffic volume, number of tracks, and a measure of visibility. However, the regression model lacked consistency with accepted a priori assumptions concerning the relationships between the study variables. The second study performed by the Armour Research Foundation was an investigation of the relationships between accidents and nine grade crossing characteristics at 7,416 locations in the State of Ohio (Crecink, 1958). A regression analysis routine was used to develop models predicting a ten-year expected accident rate. Equations were developed for four separate types of protection: painted crossbucks, reflectorized crossbucks, flashers, and gates.
In addition to the above mentioned models, there are also numerous models used by different states in the USA to prioritize rail-highway level crossings (Elzohairy & Benekohal, 2000): (1) The Department of Transportation accident prediction formula (USDOT Accident Prediction Model), (2) California’s Hazard Rating Formula, (3) Connecticut’s Hazard Rating Formula, that is very similar to that of California, (4) Kansas’s Design Hazard Rating Formula, (5) The Missouri crossing improvement program currently uses a calculated Exposure Index (Missouri’s Exposure Index Formula) to prioritize crossings for possible improvements, (6) nois’s modified expected accident frequency formula used to rank grade crossings (Elzohairy & Benekohal, 2000).

As compare to conventional cost-benefit approach, multicriteria analysis allows effective comparative evaluation among options and stakeholders over a common set of evaluation objectives. Furthermore, multi-criteria analysis could overcome the limitation of cost-benefit analysis whereby all the costs and benefits have to be expressed in monetary terms (Ćirović & Pamučar, 2013). Ford and Matthews (2002) and Roop et al. (2005) adopted multi-criteria analysis technique to assess the relative merits of the candidate protection systems and evolution of railway level crossings. In addition to classic multi-criteria techniques, Ćirović & Pamučar (2013) presented the modeling of the neuro-fuzzy system for the prioritizing of crossings. The study showed successful use of adaptive artificial intelligence models for predicting risks at the crossings.

Therefore, the managers of railway companies and agencies involved in improving road safety should try to answer several questions, such as how to prioritize level crossings and how to build a strategy of investing in the improvement of their security. In such cases, multi-criteria decision making models offer practical solutions. However, the design of multi-criteria framework for the evaluation of level crossings is a complex process that is still being developed to improve the area under consideration in this paper (Roop et al., 2005). Accordingly, in order to face the above challenges, it is necessary to develop a model for the evaluation and ranking of crossings. It is precisely this purpose that the goal of this study results from, and that is to provide a comprehensive model for making sustainable investment strategy in improving the security of crossings using multi-criteria models. In order to achieve this goal, the main research question of this study is how to form a decision-making model in which key risk indicators on the crossings are implemented and which allows determining priority of crossings while creating sustainable strategy for investing in security equipment? In order to solve this problem, this study suggests the evaluation of crossings using the FUCOM and MAIRCA models. The implementation of multi-criteria approach in the models for evaluating crossings has been very limited so far. More precisely, there are no studies that consider the integration of the FUCOM and MAIRCA models, not only in the field of evaluation of crossings, but neither in literature in the field of multi-criteria decision making (MCDM). The FUCOM-MAIRCA model is new comprehensive multi-criteria model that can be very successfully applied in other studies that are not covered by this paper.
3. Methodological presentation of the FUCOM and MAIRCA models

The FUCOM-MAIRCA model is implemented through two phases. In the first phase, through the application of the FUCOM model the expert evaluation of criteria is carried out and determining of weight coefficients of criteria. The obtained values of the weight coefficients are further used in the second phase of the model for determining the values of theoretical assessments of the MAIRCA model. In the following sections (sections 3.1 and 3.2), the steps of the FUCOM and MAIRCA model are presented in detail.

3.1. Full Consistency Method (FUCOM)

The FUCOM (Pamučar et al., 2018a) belongs to new models for subjective determining of weights of criteria in multi-criteria decision making. The FUCOM is a tool that helps managers deal with their own subjectivity in prioritizing criteria through simple algorithm and using a scale acceptable for them. Some advantages that make the authors opt for the FUCOM are the following: (1) FUCOM allows obtaining optimal weight coefficients with the ability to validate them by consistency of the results; (2) Applying FUCOM, the optimal values of weight coefficients are obtained with simple mathematical apparatus that allows favoring certain criteria in evaluating phenomena in accordance with current requirements of decision-makers and minimizing the risks in decision-making; (3) FUCOM provides optimal values of weight coefficients with minimal subjective influence and minimal impact of inconsistencies of expert preferences on the final values of the weights of criteria; (4) Only the n-1 comparison of criteria is required; (5) The model is flexible and suitable for application to different measurement scales representing expert preferences.

In the next section is presented the FUCOM algorithm including the following steps:

Step 1. In the first step, the criteria from the predefined set of the evaluation criteria \( C = \{C_1, C_2, ..., C_n\} \) are ranked. The ranking is performed according to the significance of the criteria, i.e. starting from the criterion which is expected to have the highest weight coefficient to the criterion of the least significance. Thus, the criteria ranked according to the expected values of the weight coefficients are obtained:

\[
C_{j(1)} > C_{j(2)} > ... > C_{j(k)}
\]  
(1)

where \( k \) represents the rank of the observed criterion. If there is a judgment of the existence of two or more criteria with the same significance, the sign of equality is placed instead of “>” between these criteria in the expression (1).

Step 2. In the second step, a comparison of the ranked criteria is carried out and the comparative priority \( \varphi_{k/(k+1)}, k = 1, 2, ..., n \), where \( k \) represents the rank of the criteria of the evaluation criteria is determined. The comparative priority of the evaluation criteria \( \varphi_{k/(k+1)} \) is an advantage of the criterion of the \( C_{j(k)} \) rank compared to the criterion of the \( C_{j(k+1)} \) rank. Thus, the vectors of the comparative priorities of the evaluation criteria are obtained, as in the expression (2)

\[
\Phi = (\varphi_{1/2}, \varphi_{2/3}, ..., \varphi_{n/(n+1)})
\]

(2)
where $\varphi_{k/(k+1)}$ represents the significance (priority) that the criterion of the $C_{j(k)}$ rank has compared to the criterion of the $C_{j(k+1)}$ rank.

The comparative priority of the criteria is defined in one of the two ways defined in the following part:

a) Pursuant to their preferences, decision-makers define the comparative priority $\varphi_{k/(k+1)}$ among the observed criteria. Thus, for example, if two stones A and B, which, respectively, have the weights of $w_A = 300$ grams and $w_B = 255$ grams are observed, the comparative priority ($\varphi_{A/B}$) of Stone A in relation to Stone B is $\varphi_{A/B} = 300/255 = 1.18$. Also, if the weights A and B cannot be determined precisely, but a predefined scale is used, e.g. from 1 to 9, then it can be said that stones A and B have weights $w_A = 8$ and $w_B = 7$. respectively. Then the comparative priority ($\varphi_{A/B}$) of Stone A in relation to Stone B can be determined as $\varphi_{A/B} = 8/7 = 1.14$. This means that stone A in relation to stone B has a greater priority (weight) by 1.18 (in the case of precise measurements), i.e. by 1.14 (in the case of application of measuring scale). In the same manner, decision-makers define the comparative priority among the observed criteria $\varphi_{k/(k+1)}$. When solving real problems, decision-makers compare the ranked criteria based on internal knowledge, so they determine the comparative priority $\varphi_{k/(k+1)}$ based on subjective preferences. If the decision-maker thinks that the criterion of the $C_{j(k)}$ rank has the same significance as the criterion of the $C_{j(k+1)}$ rank, then the comparative priority is $\varphi_{k/(k+1)} = 1$.

b) Based on a predefined scale for the comparison of criteria, decision-makers compare the criteria and thus determine the significance of each individual criterion in the expression (1). The comparison is made with respect to the first-ranked (the most significant) criterion. Thus, the significance of the criteria ($\sigma_{C_{j(k)}}$) for all of the criteria ranked in Step 1 is obtained. Since the first-ranked criterion is compared with itself (its significance is $\sigma_{C_{j(1)}} = 1$), a conclusion can be drawn that the n-1 comparison of the criteria should be performed.

As we can see from the example shown in Step 2b, the FUCOM model allows the pairwise comparison of the criteria by means of using integer, decimal values or the values from the predefined scale for the pairwise comparison of the criteria.

Step 3. In the third step, the final values of the weight coefficients of the evaluation criteria $(w_1, w_2, ..., w_n)^T$ are calculated. The final values of the weight coefficients should satisfy the two conditions:

a) that the ratio of the weight coefficients is equal to the comparative priority among the observed criteria $(\varphi_{k/(k+1)})$ defined in Step 2, i.e. that the following condition is met:

$$\frac{w_k}{w_{k+1}} = \varphi_{k/(k+1)}$$

(3)
b) In addition to the condition (3), the final values of the weight coefficients should satisfy the condition of mathematical transitivity, i.e. that

\[
\frac{w_k}{w_{k+1}} \otimes \frac{w_{k+1}}{w_{k+2}} = \frac{w_k}{w_{k+2}}.
\]

Since \( \varphi_{k/(k+1)} = \frac{w_k}{w_{k+1}} \) and \( \varphi_{(k+1)/(k+2)} = \frac{w_{k+1}}{w_{k+2}} \), that

\[
\frac{w_k}{w_{k+2}} = \varphi_{k/(k+1)} \otimes \varphi_{(k+1)/(k+2)}
\]

is obtained. Thus, yet another condition that the final values of the weight coefficients of the evaluation criteria need to meet is obtained, namely:

\[
\frac{w_k}{w_{k+2}} = \varphi_{k/(k+1)} \otimes \varphi_{(k+1)/(k+2)}
\] (4)

Full consistency i.e. minimum DFC (\( \chi \)) is satisfied only if transitivity is fully respected, i.e. when the conditions of

\[
\frac{w_k}{w_{k+1}} = \varphi_{k/(k+1)} \quad \text{and} \quad \frac{w_k}{w_{k+2}} = \varphi_{k/(k+1)} \otimes \varphi_{(k+1)/(k+2)}
\]

are met. In that way, the requirement for maximum consistency is fulfilled, i.e. DFC is \( \chi = 0 \) for the obtained values of the weight coefficients. In order for the conditions to be met, it is necessary that the values of the weight coefficients \( (w_1, w_2, \ldots, w_n)^T \) meet the condition of

\[
\left| \frac{w_k}{w_{k+1}} - \varphi_{k/(k+1)} \right| \leq \chi \quad \text{and} \quad \left| \frac{w_k}{w_{k+2}} - \varphi_{k/(k+1)} \otimes \varphi_{(k+1)/(k+2)} \right| \leq \chi,
\]

with the minimization of the value \( \chi \). In that manner the requirement for maximum consistency is satisfied.

Based on the defined settings, the final model for determining the final values of the weight coefficients of the evaluation criteria can be defined.

\[
\begin{align*}
\text{min} & \quad \chi \\
\text{s.t.} & \quad \left| \frac{w_{j(k)}}{w_{j(k+1)}} - \varphi_{k/(k+1)} \right| \leq \chi, \; \forall j \\
& \quad \left| \frac{w_{j(k)}}{w_{j(k+2)}} - \varphi_{k/(k+1)} \otimes \varphi_{(k+1)/(k+2)} \right| \leq \chi, \; \forall j \\
& \quad \sum_{j=1}^n w_j = 1, \; \forall j \\
& \quad w_j \geq 0, \; \forall j
\end{align*}
\] (5)

By solving the model (5), the final values of the evaluation criteria \( (w_1, w_2, \ldots, w_n)^T \) and the degree of DFC (\( \chi \)) are generated. In order to achieve a better understanding of the presented model, two simple examples will demonstrate the process of determining weight coefficients by applying FUCOM. In the first example, the procedure for determining the comparative priority (\( \varphi_{k/(k+1)} \)) is shown by applying Step 2a, whereas in the second example, \( \varphi_{k/(k+1)} \) is determined by applying Step 2b.
3.2. Multi-Attributive Ideal-Real Comparative Analysis (MAIRCA)

The basic MAIRCA set-up is to define the gap between ideal and empirical ratings (Gigović et al., 2016; Pamučar et al., 2017, 2018b; Chatterjee et al., 2018). Summing up the gap by each criterion generates the total gap for each alternative observed. Ranking the alternatives comes at the end of the process, where the best-ranked alternative is the one with the lowest gap value. The alternative with the lowest total gap value is the alternative, by most of the criteria, with the values closest to the ideal ratings (the ideal criteria values). The MAIRCA method is carried out in seven steps:

Step 1. Formulation of the initial decision-making matrix (X). The initial decision-making matrix (6) determines the criteria values (x_{ij}, i = 1,2,...,n; j = 1,2,...,m) for each alternative observed.

\[
X = \begin{bmatrix}
C_1 & C_2 & \ldots & C_n \\
A_1 & x_{11} & x_{12} & \ldots & x_{1n} \\
A_2 & x_{21} & x_{22} & \ldots & x_{2n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
A_m & x_{m1} & x_{m2} & \ldots & x_{mn}
\end{bmatrix}
\]

(6)

The criteria from the matrix (6) can be quantitative (measurable) and qualitative (descriptive). The quantitative criteria values in the matrix (6) are obtained by quantification of real indicators which present the criteria. The qualitative criteria values are determined by decision-maker's preferences or, in a case of a large number of experts, by aggregating the experts' opinions.

Step 2. Defining preferences for the choice of alternatives \( P_{A_i} \). While selecting the alternatives, the decision maker (DM) is neutral, meaning there’s no preference for any of the offered alternatives. The assumption is that the DM does not take into account the probability of choosing any particular alternative, and has no preference in the alternative selection process. The DM can then view the alternatives as if each can materialize with the same probability, and the preference for any of the m possible alternatives is

\[
P_{A_i} = \frac{1}{m}; \sum_{i=1}^{m} P_{A_i} = 1, \ i = 1,2,...,m
\]

(7)

where \( m \) is the total number of the alternatives being selected.

In a decision-making analysis with a priori probabilities we proceed from the point that the DM is neutral to selection probability of each alternative. In that case, all preferences for the selection of individual alternatives are equal, i.e.

\[
P_{A_1} = P_{A_2} = \ldots = P_{A_n}
\]

(8)

where \( m \) is the total number of the alternatives being selected.

Step 3. Calculation of the elements of the theoretical ratings matrix (\( T_p \)).
The format of the matrix \((T_p)\) is \(n \times m\) (where \(n\) is the total number of criteria, \(m\) is the total number of alternatives). The elements of the theoretical ratings matrix \((t_{ pij})\) are calculated as a product of preferences for the selection of alternatives \(P_{ Ai}\) and criterion weights \((w_i, i = 1, 2, ..., n)\)

\[
T_p = \begin{bmatrix}
    t_{p11} & t_{p12} & ... & t_{p1n} \\
    t_{p21} & t_{p22} & ... & t_{p2n} \\
    ... & ... & ... & ... \\
    t_{pm1} & t_{pm2} & ... & t_{pmn}
\end{bmatrix} = \begin{bmatrix}
    P_{ Ai} \cdot w_1 & P_{ Ai} \cdot w_2 & ... & P_{ Ai} \cdot w_n \\
    P_{ A2} \cdot w_1 & P_{ A2} \cdot w_2 & ... & P_{ A2} \cdot w_n \\
    ... & ... & ... & ... \\
    P_{ An} \cdot w_1 & P_{ An} \cdot w_2 & ... & P_{ An} \cdot w_n
\end{bmatrix}
\]

(9)

Since the DM is neutral towards the initial alternative selection, the preferences \((P_{ Ai})\) are the same for all alternatives. As the preferences \((P_{ Ai})\) are the same for all the alternatives, we can also present the matrix (9) in the format \(n \times 1\) (where \(n\) is the total number of criteria).

\[
T_p = P_{ Ai} \begin{bmatrix}
    t_{p1} \\
    t_{p2} \\
    ... \\
    t_{pm}
\end{bmatrix} = P_{ Ai} \begin{bmatrix}
    P_{ Ai} \cdot w_1 \\
    P_{ Ai} \cdot w_2 \\
    ... \\
    P_{ Ai} \cdot w_n
\end{bmatrix}
\]

(10)

where \(n\) is the total number of criteria, and \(t_{ ps}\) theoretical rating.

**Step 4. Definition of the elements of real ratings matrix \((T_r)\).**

\[
T_r = \begin{bmatrix}
    C_1 & C_2 & ... & C_n \\
    A_1 \begin{bmatrix}
    t_{r11} \\
    t_{r12} \\
    ... \\
    t_{rin}
\end{bmatrix} \\
    A_2 \begin{bmatrix}
    t_{r21} & t_{r22} & ... & t_{r2n}
\end{bmatrix} \\
    ... \\
    A_m \begin{bmatrix}
    t_{rm1} & t_{rm2} & ... & t_{rmn}
\end{bmatrix}
\end{bmatrix}
\]

(11)

where \(n\) represents the total number of criteria, and \(m\) the total number of alternatives.

In calculation of the elements of the real ratings matrix \((T_r)\) the elements of the theoretical ratings matrix \((T_p)\) are multiplied by the elements of the initial decision-making matrix \((X)\) using the following formulas:

For the benefit type criteria (preferred higher criteria value)

\[
t_{ij} = t_{ pij} \cdot \frac{x_{ij} - x_i^-}{x_i^+ - x_i^-}
\]

(12)

For the cost type criteria (preferred lower criteria value)

\[
t_{ij} = t_{ pij} \cdot \frac{x_{ij} - x_i^+}{x_i^- - x_i^+}
\]

(13)
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where $x_{ij}$, $x_i^+$ and $x_i^-$ represent the elements of the initial decision-making matrix ($X$), and $x_i^+$ i $x_i^-$ are defined as:

$$x_i^+ = \max(x_1, x_2, ..., x_m),$$

representing the maximum values of the observed criterion by alternatives.

$$x_i^- = \min(x_1, x_2, ..., x_m),$$

representing the minimum values of the observed criterion by alternatives.

**Step 5.** The calculation of the total gap matrix ($G$). The elements of the $G$ matrix are obtained as a difference (gap) between the theoretical ($t_{ij}$) and real ratings ($r_{ij}$), i.e., a difference between the theoretical ratings matrix ($T_p$) and the real ratings matrix ($T_r$).

$$G = T_p - T_r = \begin{bmatrix}
g_{11} & g_{12} & \cdots & g_{1m} 
g_{21} & g_{22} & \cdots & g_{2m} 
\vdots & \vdots & \ddots & \vdots 
g_{m1} & g_{m2} & \cdots & g_{mn}
\end{bmatrix} = \begin{bmatrix}
t_{p11} - t_{r11} & t_{p12} - t_{r12} & \cdots & t_{p1m} - t_{r1m} 
t_{p21} - t_{r21} & t_{p22} - t_{r22} & \cdots & t_{p2m} - t_{r2m} 
\vdots & \vdots & \ddots & \vdots 
t_{pm1} - t_{rm1} & t_{pm2} - t_{rm2} & \cdots & t_{pmn} - t_{rmn}
\end{bmatrix}$$

(14)

where $n$ represents the total number of criteria, $m$ is the total number of the alternatives being selected.

The gap $g_{ij}$ takes the values from the interval $g_{ij} \in [0, \infty)$, by the equation (15)

$$g_{ij} = t_{ij}$$

The preferable option is that $g_{ij}$ gravitates towards zero ($g_{ij} \to 0$), since we are choosing the alternative with the smallest difference between theoretical ratings ($t_{ij}$) and real ratings ($r_{ij}$). If for the criterion $C_i$ the alternative $A_i$ has the theoretical rating value equal to the real rating value ($t_{ij} = r_{ij}$), the gap for the alternative $A_i$, by the criterion $C_i$, is $g_{ij} = 0$. In other words, by the criterion $C_i$, the alternative $A_i$ is the best (ideal) alternative ($A_i^+$).

If by the criterion $C_i$ the alternative $A_i$ has the value of theoretical ratings $t_{ij}$, and the value of real ratings $t_{ij} = 0$, the gap for the alternative $A_i$, by the criterion $C_i$, is $g_{ij} = t_{ij}$. In other words, the alternative $A_i$ is the worst (anti-ideal) alternative ($A_i^-$) by the criterion $C_i$.

**Step 6.** The calculation of the final values of criteria functions ($Q_i$) by alternatives. The values of criteria functions are obtained by summing the gap ($g_{ij}$) by alternatives, that is, by summing the elements of matrix ($G$) by columns, Eqn. (16)
Where \( n \) is the total number of criteria, and \( m \) is the total number of the alternatives being selected.

**Step 7. Defining the dominance index** \( (A_{D,i-j}) \) **of the best-ranked alternative and final rank of alternatives.**

The dominance index of the best-ranked alternative defines its advantage in relation to the other alternatives, and determined here by applying Eqn. (17).

\[
A_{D,i-j} = \left| \frac{Q_j - Q_1}{Q_n} \right|, \quad j = 2, 3, ..., m
\]

(17)

where \( Q_1 \) denotes the criterion function of the best-ranked alternative, \( Q_n \) denotes the criterion function of the last ranked alternative, \( Q_j \) denotes the criterion function of the alternative which is compared to the best-ranked alternative, and \( m \) denotes the number of alternatives.

Once the dominance index is determined, the dominance threshold \( I_D \) is determined by applying Eqn.(18)

\[
I_D = \frac{m-1}{m^2}
\]

(18)

where \( m \) denotes the number of alternatives.

Provided that the dominance index \( A_{D,i-j} \) is greater or equal to dominance threshold \( I_D \) \( (A_{D,i-j} \geq I_D) \), the obtained rank will be retained. However, if the dominance index \( A_{D,i-j} \) is smaller than the dominance threshold \( I_D \) \( (A_{D,i-j} < I_D) \), then it cannot be said with certainty that the first ranked alternatives have an advantage over the alternative being analyzed. The said restrictions can be shown by applying the following Eqn. (19)

\[
R_{\text{final},j} = \begin{cases} 
A_{D,i-j} \geq I_D & \Rightarrow R_{\text{final},j} = R_{\text{initial},j} \\
A_{D,i-j} < I_D & \Rightarrow R_{\text{final},j} = R_{\text{initial},j}^*
\end{cases}
\]

(19)

where \( R_{\text{initial},j} \) and \( R_{\text{final},j} \) denotes the initial and final rank of the alternative, respectively, that is compared with the best-ranked alternative, \( I_D \) denotes the dominance threshold, and \( A_{D,i-j} \) denotes the dominance index of the best-ranked alternative in relation to the alternative.

Provided that criterion \( A_{D,i-j} < I_D \) is satisfied, then the rank of the alternative that is compared to the best-ranked alternative will be corrected and then treated as the best-ranked alternative and assigned the value “1*”. In this way it is emphasized that the best-ranked alternative is characterized by a smaller advantage than the one specified in Eqn. (18).
Assume, for example, that the best-ranked alternative is compared to the second-ranked alternative and that the criterion $A_{D,1-2} < I_D$ is satisfied. Then the second-ranked alternative will be assigned rank "1". The comparison may proceed with the third-ranked alternative. If for the third-ranked alternative criterion $A_{D,1-3} < I_D$ is satisfied, then the third-ranked alternative will be assigned rank "1" and so on, until reaching the last alternative.

Finally, correction of the initial ranks ($R_{initial}$) is carried out for all alternatives satisfying criterion $A_{D,1-j} < I_D$, while the ranks of alternatives satisfying the criterion $A_{D,1-j} \geq I_D$ remain unchanged. Therefore, the final rank of alternatives ($R_{final}$) which is presented simultaneously with the initial rank of alternatives ($R_{initial}$) is obtained.

4. Application of the FUCOM-MAIRCA model

The most important task of the safety management in road and rail traffic is to raise safety level of traffic at level crossings (Jankovic & Mladenovic, 2011). In order to identify the crossings which requires the intervention, either in terms of changes in safety method, or in terms of reconstruction and maintenance of road and railway infrastructure, it is necessary to dispose of various data which can be classified in three categories (Jankovic et al., 2014): (1) data on current condition of the level crossings: location of the crossing from the aspect of railway (station area or open rail) and from the aspect of road (main, regional or local), existing safety system on the level crossing, existing road and railway signalization condition in the level crossing area, type and condition of road surface at the crossing, barriers and drainage systems in the crossing area, geometric parameters of the crossing, sight triangle and distance visibility, (non) existence of opportunities for level separation, prescribed speed of trains and road vehicles in the crossings area; (2) Data on traffic accidents at crossings for a selected period: total number of accidents, structure of accidents by consequences, total number of minor, serious injuries and killed persons, total material damage and (3) Data on volume and structure of road and rail traffic at crossings.

On the basis of the recommendations from the literature (Jankovic & Mladenovic, 2011; Ćirović & Pamučar, 2013; Janković et al., 2014), as well as the empirical knowledge of four experts collected through the survey, the criteria for the evaluation of level crossings are defined and shown in the Table 1.
Table 1. Criteria for the evaluation of level crossings

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Brief description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency of rail traffic at the observed level crossing (C1) - max</td>
<td>A parameter with a major impact on the probability of occurrence of extraordinary events at road crossings. Traffic volume and frequency are influenced by trains in internal traffic, as the needs of certain regional or other centers at the part of the railway. The number of truck/month</td>
</tr>
<tr>
<td>Road traffic frequency at the observed level crossing (C2) - max</td>
<td>This factor is particularly significant in urban areas where the railway line divides city zones, where road traffic is loaded with more vehicles and pedestrians and where, due to poor technical possibilities of railway traffic equipment, waiting time for passing of a train is larger than usual (even up to 10 minutes). In urban zones, there are crossings where the road crosses several tracks. This increases the likelihood of occurrence of extraordinary events at the crossings. The number of vehicles/h</td>
</tr>
<tr>
<td>Number of tracks at the observed level crossing (C3) - max</td>
<td>The number of tracks directly affects the time that road users spend on the railroad. With the increase of the number of tracks, the time from the moment of moving of the vehicle from the stop line from one side of the crossing to the pass of the rear part of the vehicle out of the rail profile at the given crossing also increases. The number of tracks at road crossings</td>
</tr>
<tr>
<td>Maximum permitted speed of trains at the level crossing chainage (C4) - max</td>
<td>A parameter that is particularly significant for crossings that are only secured by road signs. This parameter is indirectly related to the visibility of the crossing for road vehicle drivers or pedestrians. Maximum permitted speed of trains at road section</td>
</tr>
<tr>
<td>Angle of crossing of road and rail (C5) - max</td>
<td>The optimum angle of crossing of rail and road at the crossing is 90 degrees. However, the construction possibilities, the terrain configuration, the position of the existing roads and other circumstances make the road and rail crossing angle in practice range from 30 to 175 degrees. Angle of crossing of road and rail</td>
</tr>
<tr>
<td>Number of extraordinary events at the observed level crossing (C6) - max</td>
<td>Extraordinary events are followed by great material damage, killed and severely injured persons. Number of extraordinary events at level crossing</td>
</tr>
<tr>
<td>Sight of the observed crossing from the aspect of road traffic (C7) - min</td>
<td>Sight at a given road crossing is a parameter that has an impact on the decision of a road vehicle driver to start driving over the crossing in cases where the crossing is not secured by active protection devices (semi-barriers or barriers). Sight of the crossing means that when a driver stops his vehicle on the stop line, he can observe the traffic situation. The qualitative criterion that evaluating using linguistic scale 1 - 9</td>
</tr>
</tbody>
</table>
As defined in the previous section, the first phase of the model implies the application of the FUCOM to determine weight coefficients of criteria.

Step 1. In the first step, the criteria are ranged from the defined set of criteria, which is shown in the Table 1. Ranking of the criteria according to its significance is carried out by four experts.

Table 2. Rank of criteria

<table>
<thead>
<tr>
<th>Expert</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>C2&gt;C5&gt;C7&gt;C1&gt;C6&gt;C3&gt;C4</td>
</tr>
<tr>
<td>E2</td>
<td>C2&gt;C5&gt;C7&gt;C1=C6&gt;C3&gt;C4</td>
</tr>
<tr>
<td>E3</td>
<td>C2&gt;C5&gt;C7&gt;C1&gt;C6&gt;C4&gt;C3</td>
</tr>
<tr>
<td>E4</td>
<td>C2&gt;C7&gt;C5&gt;C1=C6&gt;C3&gt;C4</td>
</tr>
</tbody>
</table>

Step 2. In the second step, comparison of the ranked criteria is done and comparative significance of the evaluation criteria is determined. Comparative significance of the evaluation criteria is obtained by the survey of experts and it is shown in the Table 3.

Table 3. Comparative significance of criteria

<table>
<thead>
<tr>
<th>Expert</th>
<th>Comparative Significance ( \varphi_{k/(k+1)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>C2</td>
</tr>
<tr>
<td></td>
<td>C5</td>
</tr>
<tr>
<td></td>
<td>C7</td>
</tr>
<tr>
<td></td>
<td>C1</td>
</tr>
<tr>
<td></td>
<td>C6</td>
</tr>
<tr>
<td></td>
<td>C3</td>
</tr>
<tr>
<td></td>
<td>C4</td>
</tr>
<tr>
<td>E2</td>
<td>C2</td>
</tr>
<tr>
<td></td>
<td>C5</td>
</tr>
<tr>
<td></td>
<td>C7</td>
</tr>
<tr>
<td></td>
<td>C1</td>
</tr>
<tr>
<td></td>
<td>C6</td>
</tr>
<tr>
<td></td>
<td>C3</td>
</tr>
<tr>
<td></td>
<td>C4</td>
</tr>
</tbody>
</table>

Step 3. In this step, the final values of the weight coefficients of the evaluation criteria are calculated \( (w_1, w_2, ..., w_7)^T \) forming the model (5). By applying both the expressions (3) and (4) and the data from the Table 3, it is formed special model for determining the weight coefficients of the criteria for every expert:
By solving the presented models in the Lingo 17.0 software, we obtain the weight coefficients of the criteria for every expert, as shown in Table 4.

<table>
<thead>
<tr>
<th>Expert</th>
<th>$w_1$</th>
<th>$w_2$</th>
<th>$w_3$</th>
<th>$w_4$</th>
<th>$w_5$</th>
<th>$w_6$</th>
<th>$w_7$</th>
<th>DFC (X)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>0.1318</td>
<td>0.2190</td>
<td>0.1141</td>
<td>0.0827</td>
<td>0.1711</td>
<td>0.1256</td>
<td>0.1556</td>
<td>0.0000</td>
</tr>
<tr>
<td>E2</td>
<td>0.1319</td>
<td>0.2145</td>
<td>0.1147</td>
<td>0.0917</td>
<td>0.1638</td>
<td>0.1319</td>
<td>0.1516</td>
<td>0.0000</td>
</tr>
<tr>
<td>E3</td>
<td>0.1294</td>
<td>0.2140</td>
<td>0.0910</td>
<td>0.1093</td>
<td>0.1754</td>
<td>0.1256</td>
<td>0.1553</td>
<td>0.0002</td>
</tr>
<tr>
<td>E4</td>
<td>0.1327</td>
<td>0.2051</td>
<td>0.1134</td>
<td>0.0872</td>
<td>0.1552</td>
<td>0.1326</td>
<td>0.1738</td>
<td>0.0002</td>
</tr>
<tr>
<td>Average</td>
<td>0.1314</td>
<td>0.2132</td>
<td>0.1083</td>
<td>0.0927</td>
<td>0.1664</td>
<td>0.1289</td>
<td>0.1591</td>
<td>-</td>
</tr>
</tbody>
</table>

From the Table 4, it can be observed that the FUCOM provides fully consistent values of weight coefficients, since for every of the four models DFC=0. Final values of the weight coefficients are obtained by averaging the weights obtained from every of the four models shown.

After calculating the weight coefficients of the criteria ($w_j$), the evaluation of the crossings is carried out using the MAIRCA method. In the Table 5 are shown the characteristics of ten level crossings (alternatives). The evaluation of the qualitative criteria C7 is made based on the assessments of the observed level crossing changing through the nine-degree scale.
After forming the initial decision matrix, as in the Table 5, the preferences are made according to the selection of the alternatives $P_{A_i}$. Since during the evaluation of the level crossing, experts do not have clear preference for selecting certain alternatives, then $P_{A_i}$ is determined by applying the expression (7)

$$P_{A_i} = \frac{1}{m} = \frac{1}{10} = 0.10$$

In this case, all preferences for the selection of certain alternatives are the same (8)

$$P_{A_1} = P_{A_2} = ... = P_{A_{10}} = 0.10$$

The calculation of the elements of the matrix of theoretical assessments ($T_p$), from the Table 6, is performed using the expression (9), respectively (10). Matrix elements are calculated by multiplying the preferences of selected alternatives $P_{A_i}$ and the weight coefficients of criteria ($w_i$).

Table 6. Matrix of theoretical weights $T_p$

<table>
<thead>
<tr>
<th>Alternative</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0.0131</td>
<td>0.0213</td>
<td>0.0108</td>
<td>0.0093</td>
<td>0.0166</td>
<td>0.0129</td>
<td>0.0159</td>
</tr>
<tr>
<td>A2</td>
<td>0.0131</td>
<td>0.0213</td>
<td>0.0108</td>
<td>0.0093</td>
<td>0.0166</td>
<td>0.0129</td>
<td>0.0159</td>
</tr>
<tr>
<td>A3</td>
<td>0.0131</td>
<td>0.0213</td>
<td>0.0108</td>
<td>0.0093</td>
<td>0.0166</td>
<td>0.0129</td>
<td>0.0159</td>
</tr>
<tr>
<td>A4</td>
<td>0.0131</td>
<td>0.0213</td>
<td>0.0108</td>
<td>0.0093</td>
<td>0.0166</td>
<td>0.0129</td>
<td>0.0159</td>
</tr>
<tr>
<td>A5</td>
<td>0.0131</td>
<td>0.0213</td>
<td>0.0108</td>
<td>0.0093</td>
<td>0.0166</td>
<td>0.0129</td>
<td>0.0159</td>
</tr>
<tr>
<td>A6</td>
<td>0.0131</td>
<td>0.0213</td>
<td>0.0108</td>
<td>0.0093</td>
<td>0.0166</td>
<td>0.0129</td>
<td>0.0159</td>
</tr>
<tr>
<td>A7</td>
<td>0.0131</td>
<td>0.0213</td>
<td>0.0108</td>
<td>0.0093</td>
<td>0.0166</td>
<td>0.0129</td>
<td>0.0159</td>
</tr>
<tr>
<td>A8</td>
<td>0.0131</td>
<td>0.0213</td>
<td>0.0108</td>
<td>0.0093</td>
<td>0.0166</td>
<td>0.0129</td>
<td>0.0159</td>
</tr>
<tr>
<td>A9</td>
<td>0.0131</td>
<td>0.0213</td>
<td>0.0108</td>
<td>0.0093</td>
<td>0.0166</td>
<td>0.0129</td>
<td>0.0159</td>
</tr>
<tr>
<td>A10</td>
<td>0.0131</td>
<td>0.0213</td>
<td>0.0108</td>
<td>0.0093</td>
<td>0.0166</td>
<td>0.0129</td>
<td>0.0159</td>
</tr>
</tbody>
</table>
After forming the matrix of theoretical assessments ($T_p$), it is calculated the matrix of real assessments ($T_r$). The calculation of the real assessment matrix elements (Table 7) is carried out by multiplying the elements of the matrix of theoretical assessment ($T_p$) and normalized elements of the initial decision making matrix. Normalization of elements of the initial decision making matrix is performed using the expressions (12) and (13).

Table 7. Matrix of real assessments

<table>
<thead>
<tr>
<th>Alternative</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0.0044</td>
<td>0.0204</td>
<td>0.0054</td>
<td>0.0041</td>
<td>0.0104</td>
<td>0.0129</td>
<td>0.0159</td>
</tr>
<tr>
<td>A2</td>
<td>0.0096</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0022</td>
<td>0.0166</td>
<td>0.0052</td>
<td>0.0023</td>
</tr>
<tr>
<td>A3</td>
<td>0.0000</td>
<td>0.0213</td>
<td>0.0054</td>
<td>0.0011</td>
<td>0.0077</td>
<td>0.0077</td>
<td>0.0136</td>
</tr>
<tr>
<td>A4</td>
<td>0.0110</td>
<td>0.0094</td>
<td>0.0000</td>
<td>0.0082</td>
<td>0.0063</td>
<td>0.0026</td>
<td>0.0159</td>
</tr>
<tr>
<td>A5</td>
<td>0.0067</td>
<td>0.0156</td>
<td>0.0000</td>
<td>0.0014</td>
<td>0.0002</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>A6</td>
<td>0.0088</td>
<td>0.0000</td>
<td>0.0054</td>
<td>0.0008</td>
<td>0.0147</td>
<td>0.0103</td>
<td>0.0068</td>
</tr>
<tr>
<td>A7</td>
<td>0.0033</td>
<td>0.0129</td>
<td>0.0054</td>
<td>0.0079</td>
<td>0.0104</td>
<td>0.0129</td>
<td>0.0000</td>
</tr>
<tr>
<td>A8</td>
<td>0.0131</td>
<td>0.0100</td>
<td>0.0108</td>
<td>0.0082</td>
<td>0.0011</td>
<td>0.0103</td>
<td>0.0091</td>
</tr>
<tr>
<td>A9</td>
<td>0.0028</td>
<td>0.0045</td>
<td>0.0108</td>
<td>0.0093</td>
<td>0.0000</td>
<td>0.0077</td>
<td>0.0000</td>
</tr>
<tr>
<td>A10</td>
<td>0.0072</td>
<td>0.0095</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0159</td>
<td>0.0077</td>
<td>0.0136</td>
</tr>
</tbody>
</table>

The elements of the total gap matrix ($G$) are obtained as the difference (gap) between theoretical ($t_{pi}$) and real assessments ($t_{rij}$), respectively, by subtracting the elements of the matrix of theoretical assessments ($T_p$) and the elements of the real assessment matrix ($T_r$). By applying the expression (14) we obtain final total gap matrix, as shown in the Table 8. It is desirable that the value $g_{ij}$ tends to zero ($g_{ij} \to 0$), since we select the alternative with the slightest difference between theoretical ($t_{pi}$) and real assessments ($t_{rij}$).

Table 8. Total gap matrix

<table>
<thead>
<tr>
<th>Alternative</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0.0088</td>
<td>0.0009</td>
<td>0.0054</td>
<td>0.0052</td>
<td>0.0063</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>A2</td>
<td>0.0035</td>
<td>0.0213</td>
<td>0.0108</td>
<td>0.0071</td>
<td>0.0000</td>
<td>0.0077</td>
<td>0.0136</td>
</tr>
<tr>
<td>A3</td>
<td>0.0131</td>
<td>0.0000</td>
<td>0.0054</td>
<td>0.0082</td>
<td>0.0089</td>
<td>0.0052</td>
<td>0.0023</td>
</tr>
<tr>
<td>A4</td>
<td>0.0021</td>
<td>0.0119</td>
<td>0.0108</td>
<td>0.0011</td>
<td>0.0104</td>
<td>0.0103</td>
<td>0.0000</td>
</tr>
<tr>
<td>A5</td>
<td>0.0065</td>
<td>0.0057</td>
<td>0.0108</td>
<td>0.0079</td>
<td>0.0164</td>
<td>0.0129</td>
<td>0.0159</td>
</tr>
<tr>
<td>A6</td>
<td>0.0044</td>
<td>0.0213</td>
<td>0.0054</td>
<td>0.0085</td>
<td>0.0019</td>
<td>0.0026</td>
<td>0.0091</td>
</tr>
<tr>
<td>A7</td>
<td>0.0098</td>
<td>0.0084</td>
<td>0.0054</td>
<td>0.0014</td>
<td>0.0063</td>
<td>0.0000</td>
<td>0.0159</td>
</tr>
<tr>
<td>A8</td>
<td>0.0000</td>
<td>0.0113</td>
<td>0.0000</td>
<td>0.0027</td>
<td>0.0128</td>
<td>0.0103</td>
<td>0.0114</td>
</tr>
<tr>
<td>A9</td>
<td>0.0103</td>
<td>0.0168</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0166</td>
<td>0.0052</td>
<td>0.0068</td>
</tr>
<tr>
<td>A10</td>
<td>0.0060</td>
<td>0.0118</td>
<td>0.0108</td>
<td>0.0093</td>
<td>0.0007</td>
<td>0.0052</td>
<td>0.0023</td>
</tr>
</tbody>
</table>

The values of the criteria functions ($Q_i$) by alternatives (Table 9) are obtained by summing the gap ($g_{ij}$) by alternatives, as in the expression (16).
Multi-criteria FUCOM-MAIRCA model for the evaluation of level crossings: Case study in the Republic of Serbia

Table 9. Ranking alternatives according to the MAIRCA method

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Q</th>
<th>R_{initial}</th>
<th>R_{final}</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0.0266</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>A2</td>
<td>0.0641</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>A3</td>
<td>0.0431</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>A4</td>
<td>0.0466</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>A5</td>
<td>0.0761</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>A6</td>
<td>0.0532</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>A7</td>
<td>0.0472</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>A8</td>
<td>0.0485</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>A9</td>
<td>0.0557</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>A10</td>
<td>0.0460</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

A1>A3>A10>A4>A7>A8>A6>A9>A2>A5

Based on the obtained values of the criteria functions \( Q \) is determined the initial rank of alternatives \( R_{initial} \). According to the initial ranking, the best-ranked alternative is the alternative A1. In order to conclude whether the A1 is also the best alternative, it is necessary to determine if it sufficiently dominates over the other alternatives. It is therefore necessary to determine the index of domination of the alternative A1 \( A_{D,A1-j} \) over the other alternatives, as in the expression (17). Before determining the index of domination \( A_{D,A1-j} \), using the expression (16), the dominance threshold \( I_D \) is to be defined which must be met by the alternative A1 so as to be ranked as the first one in final ranking.

\[
I_D = \frac{n - 1}{n^2} = \frac{10 - 1}{10^2} = 0.090
\]

Since the condition \( A_{D,A1-j} \geq I_D \) is fulfilled for all the alternatives, we can conclude that all initial ranks of the alternatives are retained, respectively, that \( R_{initial} = R_{final} \), as shown in the Table 9. On the basis of the obtained results, we conclude that the A1 alternative is first-ranked, respectively, A1> A3> A10> A4> A7> A8> A6> A9> A2> A5.

5. Sensitivity analysis and validation of results

The results of the multi-criteria models can significantly be influenced by the values of weight coefficients of the evaluation criteria. That is why the analysis of the influence of altering weight coefficients on the results of the research is a logical step to test the robustness of the applied model and the obtained results. Therefore, in this part of the paper is carried out the sensitivity analysis of the ranks of alternatives to changes in weight coefficients of the criteria. The sensitivity analysis is performed through seven situations. In every situation, one criterion is favorized whose weight coefficient is increased by 50 %. In the same situation, the weight coefficients are
reduced by 50% in the remaining criteria. Changes in the ranks of alternatives in seven situations are shown in the Figure 1.

The results (Figure 1) show that assigning different weights of criteria through situations leads to minor variations in the ranking of alternatives, which confirms that the model is sensitive to changes in weight coefficients. By comparing the first-ranked alternatives (A1 and A3), we note that the alternative A1 retains its rank in all situations (it remained the first-ranked), while the alternative A3 in five situations keeps its ranking, and in two situations it is third-ranked. During sensitivity analysis there was a change of ranks of the alternatives A2, A9 and A6. However, we can conclude that these changes were not drastic, as evidenced by high rank correlation through situations (Figure 2). The correlation was determined using Spearman's coefficient of correlation (Chatterjee et al, 2018).

Figure 1. Sensitivity analysis of the ranks of alternatives through seven situations

Figure 2. Correlation of ranks through seven situations of sensitivity analysis
The values of Spearman’s coefficient of correlation were obtained by comparing the initial rank of the FUCOM-MAIRCA model (Table 9) with the ranks obtained through the situations (Figure 1). In the Figure 2, we note that there is extremely high correlation of ranks, since in all situations the value of the correlation coefficient is higher than 0.970. Mean value of the correlation coefficient through all the situations amounts to 0.990, which shows extremely high correlation. Since all values of the correlation coefficient are significantly greater than 0.90, we can conclude that there is a very high correlation (closeness) of ranks and that the proposed ranking is confirmed and credible.

6. Conclusion

In this research is presented the use of multi-criteria FUCOM-MAIRCA model for evaluating level crossings. The key contribution of this paper is new FUCOM-MAIRCA model for the evaluation of crossings. Presented model allows consideration of subjectivity in the process of group decision making through linguistic evaluation of the evaluation criteria. In addition, the model presented in this paper introduces new methodological principles for the evaluation of the crossings, which at the same time contributes to the improvement of theoretical basis of multi-criteria decision making in general. The developed approach allows bridging the gap that currently exists in the methodology for evaluating the crossings.

The FUCOM-MAIRCA model was applied in the evaluation of ten level crossings on the territory of the Republic of Serbia. The results obtained were verified through sensitivity analysis carried out based on seven situations. The stability of the model is verified through statistical correlation coefficient showing high correlation of ranks in all situations. Consideration of the results and sensitivity analysis of the FUCOM-MAIRCA model show significant stability of the results and promising applicability of the model shown. Securing level crossings represents significant material expenditure, so it is necessary to be grounded on reliable strategies for the selection of a level crossing that needs to be secured, as well as to be supported by the investments realization plan in terms of its security. Also, this integrated FUCOM-MAIRCA model can be applied for evaluation of reliable strategies for the selection of a level crossing that needs to be secured in the next phase.

Since these are new models of multi-criteria decision making, the directions of future research should focus on the application of uncertainty theories (fuzzy sets, rough numbers, gray numbers, neutrosophic sets etc.) in the FUCOM and MAIRCA models. The integration of the uncertainty theories in the FUCOM and MAIRCA models would allow significant exploitation of uncertainty and subjectivity existing in the decision-making process.

References


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